THE A. C. CONDUCTIVITY OF A PLASMA

by

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Abstract

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This paper computes the electrical conductivity of a fully ionized, spatially homogeneous plasma under the influence of a uniform, periodically alternating electric field. The velocity distribution of the electrons is determined by solving the linearized Fokker-Planck equations. All the terms in the collision integral are retained, including those representing electron-electron interactions. The resultant values of conductivity is expected to be valid in the range of frequencies from zero to below the plasma frequency.

Author

I. Introduction

The purpose of this paper is to calculate the A.C. conductivity of a spatially homogeneous plasma using the Fokker-Planck Equation. The D.C. conductivity of a plasma has been calculated in the well-known works of Cohen, Spitzer and Routly,[1] and Spitzer and Harm[2]. Their results are in good agreement with the later experimental works of Lin, et al[3]. Bernstein and Trehan compute the A.C. conductivity assuming a Lorentz gas model[4]. The A.C. conductivity of a real gas should approach that of a Lorentz gas at high frequencies. (See detailed discussions in Section IV.) Toward lower frequencies their departure is expected to increase so that their ratio becomes nearly 2 in the D.C. limit, in accordance with References (1) and (2). The most recent works on A.C. conductivity by Dawson, et al.,[5],[6] consider the time variation of the two-particle distribution, which is necessary when dealing with A.C. currents of ultra-high frequencies. However, the domain of applicability of their work is limited to the frequency range $\omega > \omega_p$, where ω_p denotes the plasma frequency. Thus, a more precise calculation for the low and intermediate range of ω appears desirable.

The basic equation to be used in the present work is the Boltzmann-Fokker-Planck Equation:

$$\frac{\partial f_i}{\partial t} + \vec{v}\cdot\frac{\partial f_i}{\partial \vec{r}} + \frac{\vec{F}\partial f_i}{m\partial \vec{v}} = \left(\frac{\delta f_i}{\delta t}\right)_c \qquad (1)$$

where f_i is the distribution function of particles of type i, $\left(\frac{\delta f_i}{\delta t}\right)_c$ is the change of f_i produced by collisions.

Equation (1) is deduced from Liouville theorem to describe a many-particle system under two assumptions:

1) That the characteristic dimensions of the inhomogeneities are much larger than the average impact parameter for the particles participating in the collision.

2) That the characteristic time variation of the process is much larger than the duration of an average collision, or in other words, a collision is completed and the correlation function is "relaxed" before the distribution function itself makes any appreciable change.

It should be noted here that the term "duration of collision" is different from the so-called "collision time"; collision time is the time between two collisions. For particles interacting through long range forces, this time may be regarded as the time in which deflections gradually deflected the considered particle by 90°. Duration of collision is the time during which an interaction takes place. In a plasma it is of the order ω_p^{-1}. In Figure 1 a time scale diagram is drawn, and the validity of our calculation and those of Dawson, Oberman, and Ron are indicated.

The explicit expression of $\left(\frac{\partial f_i}{\partial t}\right)_c$ depends on the nature of the interaction force. In a fully ionized plasma, the particles

interact through the long range Coulomb forces. The cumulative effect of "weak" deflections resulting from the relatively distant collisions outweighs the effect of occasional large deflections due to relatively close collisions, so one may neglect the contribution by those very close encounters[1] - encounters which result in deflections of 90° or larger.

Also, the effect of distant particles lying outside of the Debye length λ_D may be neglected because of the shielding of inner particles. Thus, in the computations of $\left(\frac{\partial f_i}{\partial t}\right)_c$, it is only necessary to consider the collisions with impact distance intermediate between λ_D and b_o, where $b_o = \frac{e^2}{KT}$ is the impact parameter yielding a 90° deflection. The effects of these collisions are cumulative, and the total deflection produced in an interval of time is similar to that of the Brownian motion; hence, one may expand $\left(\frac{\partial f_i}{\partial t}\right)_c$ in powers of $\langle\Delta\vec{v}\rangle$, where $\langle\Delta\vec{v}\rangle$ is the average velocity change due to collisions.[1],[7] This procedure leads to the following Fokker-Planck collision integral:[8]

$$\left(\frac{\partial f_i}{\partial t}\right)_c = \Gamma_i\left\{\frac{\partial}{\partial\vec{v}}\left(f_i\frac{\partial h_i}{\partial\vec{v}}\right) + \frac{1}{2}\frac{\partial^2}{\partial\vec{v}\partial\vec{v}}\left(f_i\frac{\partial^2 g}{\partial\vec{v}\partial\vec{v}}\right)\right\} \qquad (2)$$

where

$$h_i = \sum_j \frac{m_i+m_j}{m_j}\int d\vec{v}' f_j(\vec{v}')|\vec{v}-\vec{v}'|^{-1} \qquad (3)$$

$$g = \sum_j \int d\vec{v}' f_j(\vec{v}') |\vec{v} - \vec{v}'| \tag{4}$$

and

$$\Gamma_i = \frac{4\pi z^2 e^4}{m_i^2} \ln \frac{m_i m_j v_{th}^2 \lambda_D}{2(m_i + m_j) e^2} \tag{5}$$

The summation in h_i and g sums over all species, m_i is the mass of the "i"th species, e is the electronic charge, $\lambda_D = \left(\frac{KT}{4\pi n e^2}\right)^{\frac{1}{2}}$ is the Debye length.

In this paper we consider only plasma with singly-charged ions. The extension of the present method to those with multiply-charged ions is straight-forward.

II. Derivation of Equations and Formulae

If the distribution function f has an azimuthal symmetry about a certain axis, then, following Rosenbluth, MacDonald, and Judd[8], the collision term may be written down explicitly in spherical polar coordinates in velocity space:

$$\begin{aligned}
\left(\frac{\partial f_i}{\partial t}\right)_c = \Gamma_i \Bigg\{ & -v^{-2} \frac{\partial}{\partial v}\left[f_i v^2 \frac{\partial h_i}{\partial v}\right] - v^{-2} \frac{\partial}{\partial \mu}\left[f_i (1-\mu^2) \frac{\partial h_i}{\partial \mu}\right] + (2v^2)^{-1} \frac{\partial^2}{\partial v^2}\left[f_i v^2 \frac{\partial^2 g}{\partial v^2}\right] \\
& + (2v^2)^{-1} \frac{\partial^2}{\partial \mu^2}\left[f_i \left\{v^{-2}(1-\mu^2)^2 \frac{\partial g^2}{\partial \mu^2} + v^{-1}(1-\mu^2)\frac{\partial g}{\partial v} - v^{-2}\mu(1-\mu^2)\frac{\partial g}{\partial \mu}\right\}\right] \\
& + v^{-2} \frac{\partial^2}{\partial \mu \partial v}\left[f_i (1-\mu^2)\left\{\frac{\partial^2 g}{\partial \mu \partial v} - v^{-1}\frac{\partial g}{\partial \mu}\right\}\right] \\
& + (2v^2)^{-1} \frac{\partial}{\partial v}\left[f_i \left\{-v^{-1}(1-\mu^2)\frac{\partial^2 g}{\partial \mu^2} - 2\frac{\partial g}{\partial v} + 2\mu v^{-1}\frac{\partial g}{\partial \mu}\right\}\right] \\
& + (2v^2)^{-1} \frac{\partial}{\partial \mu}\left[f_i \left\{v^{-2}\mu(1-\mu^2)\frac{\partial^2 g}{\partial \mu^2} + 2\mu v^{-1}\frac{\partial g}{\partial v} + 2v^{-1}(1-\mu^2)\frac{\partial^2 g}{\partial \mu \partial v} - 2v^{-2}\frac{\partial g}{\partial \mu}\right\}\right]\Bigg\}
\end{aligned} \tag{6}$$

where $\mu = \cos\theta$ is the direction cosine between $\vec{v}$ and $\vec{E}$. Equation (6) is an exact expression of the Fokker-Planck Equation in spherical coordinates for a distribution function with azimuthal symmetry.

We assume that the system is subject to a weak electric field $\vec{E}_o e^{i\omega t}$ whose direction lies along z-axis. Then following Chapman[9] Cowling,[10] and Spitzer,[1] we expand f_i in a power series of E:

$$f_i(\vec{v},t) = f_i^{(o)}(v) + E_o f_i^{(1)}(\vec{v},t) + E_o^2 f_i^{(2)}(\vec{v},t) + \ldots\ldots \quad (7)$$

where $f_i^{(o)}(v)$ is a time independent Maxwellian distribution and $f_i^{(1)}(\vec{v},t)$, $f_i^{(2)}(\vec{v},t)$, ... are the perturbed part due to applied electric field. When a steady state has been reached and no transient current exists, the time dependent part of $f_i^{(j)}(\vec{v},t)$ must be proportional to $e^{i\omega t}$. Since the average energy imparted to the electrons between encounters is small compared with their kinetic energy, the velocity dependent part of $f_i^{(j)}(\vec{v},t)$ can be written as $e^{-\frac{m_i}{2KT}v^2} D_i^{(j)}(v)\mu$. Therefore, we have

$$f_i^{(j)}(\vec{v},t) = \frac{m^{3/2}}{(2\pi KT)^{3/2}} e^{-\frac{m_i v^2}{2KT}} D_i^{(j)}(v)\mu e^{i\omega t} \quad (8)$$

Combining equations (1), (6), and (7) keeping only terms linear in E_o, we obtain

$$\frac{\partial f_i^{(1)}}{\partial t} + \frac{e}{m_i} e^{i\omega t} \vec{E}_o \frac{\partial}{\partial \vec{v}} f_i^{(o)} = \left(\frac{\partial f_i^{(1)}}{\partial t}\right)_c \quad (9)$$

where $\left(\frac{\partial f_i}{\partial t}\right)_c^{(1)}$ is the linearized Fokker-Planck collision integral.

Since the ions' contribution to electric current is negligible compared to electrons, we will consider only electron distributions and drop the subscript i in the distribution function hereafter.

Substituting Equation (8) into Equation (9) we find, after some algebraic manipulations, the following second order linear integral-differential equation:

$$D''(x) + P(x)D'(x) + Q(x)D(x) = R(x) + S(x) \tag{10}$$

where

$$P(x) = -2x - \frac{1}{x} + \frac{2x^2\Phi'(x)}{H(x)} \tag{11}$$

$$Q(x) = \frac{-iBx^3 + 2(1 + \Phi - 2x^3\Phi')}{H(x)} + \frac{1}{x^2} \tag{12}$$

$$R(x) = -\frac{2\alpha x^4}{H(x)} - \frac{8(2.4x^6-2x^4)}{3\pi^{\frac{1}{2}}H(x)} I_o(\infty) \tag{13}$$

$$S(x) = \frac{16}{3\pi^{\frac{1}{2}}H(x)}\left\{xI_3(x) - 1.2xI_5(x) - x^4I_o(x)(1-1.2x^2)\right\} \tag{14}$$

$$\Phi(x) = \int_o^x e^{-y^2}\,dy \tag{15}$$

$$H(x) = \Phi(x) - x\Phi'(x) \tag{16}$$

$$I_n(x) = \int_o^x y^n D(y) e^{-y^2}\,dy \tag{17}$$

$$I_o(\infty) = \int_o^\infty D(y) e^{-y^2}\,dy \tag{18}$$

with $\alpha = -E_oKT/\pi e^3 n \ln\lambda$, $\lambda = \lambda_D/b_o$

$x = v/(\frac{2KT}{m})^{\frac{1}{2}}$, $B = \frac{4\sqrt{2}\lambda}{\omega_p \ln\lambda} \cong \frac{\omega}{\omega_c}$,

where ω_c is approximately the 90^o deflection time of a particle with thermal velocity. When $\omega = 0$, Equation (10)* reduces

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* Note that $I_o(\infty)$ is essentially the total change of momentum of electrons arising from electron-ion interactions.

(Continued on next page)

to Equation (8) of Reference (8) which considers D. C. electric conductivity.

In a D. C. electric field, the electrons are not accelerated in a steady state. Hence, the inertia force term is zero and $I_o(\infty) = \frac{3\pi^{\frac{1}{2}}}{8}\alpha$.

III. Solution of Equation

Equation (10) is a linear integral-differential equation whose unknown D(x) is a complex function of a real variable. The present section will discuss the method of its solution. As will be evident in what follows, the procedure for numerical integration is far from straight-forward.

On the one hand, we encounter the problem of the instability of the solution at small and at large x. Because of the existence of singularities in Equation (10) at x = 0 and at x = ∞, a slight deviation of D(x) at either small or large x, tends to be built up quite rapidly. In order to obtain a physically acceptable solution, it is required that D(x) does not approach infinity too fast, leading to infinite conductivities. The starting value of D at small x can be obtained by means of a series of solutions.

Since the mutual electronic interaction cannot change the total momentum of the electrons, $I_o(\infty)$, by Newton's second law, must equal the total force exerted on the electrons by the applied field minus the inertia force of electrons. This relation gives us:

$$I_o(\infty) = \frac{3\pi^{\frac{1}{2}}}{8}\alpha - \frac{iB}{2}I_3(\infty) \quad .$$

But because of the instability, we cannot proceed to integrate in a step wise manner. To overcome this difficulty, we adopted a scheme used by Cohen, Spitzer, and Routly.[1] We shall refer to their paper (Reference (1)) for full details.

On the other hand, we note that $I_o(\infty)$ is no longer a known quantity as it is in the case of D. C. conductivity; it depends on the solution D(x) itself. We proceed as follows. Since we want conductivities at different frequencies, it is necessary to obtain solutions for different values of the parameter B. We begin with a small value B = 0.05. Using an $I_o(\infty)$ taken from the D. C. case, i.e. $I_o(\infty) = 0.665$, we obtain a solution to Equation (10) from which we get a new $I_o(\infty)$. Next, we pass on to B = 0.1 using the $I_o(\infty)$ obtained for the previous B. In this way, we proceed to ever increasing values of B, until the initial adopted $I_o(\infty)$ and the final calculated $I_o(\infty)$ differ by no more than 2 per cent. This occurs at B = 1.37. From this point on, we resort to a method of systematic trials. The initial and final $I_o(\infty)$ for all values of B agree to within 2 per cent, which is considered sufficiently accurate for the present purposes.

IV. Results and Discussion

The current is given by

$$\vec{J} = -e\int d\vec{v}\,\vec{v}\,f_1(\vec{v},t) = A\vec{E}_o e^{i\omega t} I_3(\infty) \qquad (19)$$

where $$I_3'(\infty) = \int_0^\infty x^3 D(x) e^{-x^2}\, dx$$

$$A = \frac{2}{3} \frac{(2KT)^{3/2}}{\pi^{3/2} m^{1/2} e^2 \ln\lambda} \tag{20}$$

Since $\underset{\sim}{J} = \sigma \underset{\sim}{E}$, we have the complex conductivity

$$\sigma = A I_3(\infty) \quad , \tag{21}$$

the impedance $$Z = \frac{1}{A I_3(\infty)} = \frac{c}{I_3(\infty)} \quad , \tag{22}$$

the resistance $$R = \frac{1}{A} \frac{\mathrm{Re} I_3(\infty)}{|I_3(\infty)|^2} = \frac{c\,\mathrm{Re} I_3(\infty)}{|I_3(\infty)|^2} \quad , \tag{23}$$

and the reactance $$X = \frac{1}{A} \frac{\mathrm{Im} I_3(\infty)}{|I_3(\infty)|^2} = \frac{c\,\mathrm{Im} I_3(\infty)}{|I_3(\infty)|^2} \quad , \tag{24}$$

with $$c = \frac{1}{A} = \frac{3}{2} \frac{1}{(2KT)^{3/2}} \pi^{3/2} m^{1/2} e^2 \ln\lambda \quad .$$

It may be remarked here that the A. C. conductivity depends on three factors:

1) The inertia of the conducting electrons.

2) The mutual interaction among electrons and ions.

3) The mutual interaction among electrons themselves.

The mutual electronic interactions have no direct effect on conductivity since the total change of momentum due to such interactions is zero. Nevertheless, they alter the distribution of electrons and thereby modify the effect which electron-ion collisions and electron inertia have in impeding the current. When

ω is small, the conductivity is primarily determined by collisions. The inclusion of electron-electron interactions reduces the conductivity by a factor of approximately two. As ω increases and becomes of order ω_c, this effect becomes less and less important because there is then insufficient time in each A. C. cycle to allow an effective modification of the distribution by electron-electro interactions. When ω well exceeds ω_c, we may neglect this effect and D(x) reduces to

$$D_{\ell}(x) = \frac{x^4}{1 + \frac{iB}{2}x^3} \tag{25}$$

and the corresponding conductivity becomes

$$\sigma_{\ell}(x) = A\int_0^{\infty} \frac{x^7 e^{-x^2}}{1 + \frac{iB}{2}x^3}\, dx \tag{26}$$

which is just the A. C. conductivity of a Lorentz gas.[4]

If we further increase ω, the inertia electrons become dominant. Then we may treat collision effect as a perturbation and obtain

$$\sigma_{\infty} = A\left(-\frac{3\pi^{\frac{1}{2}}}{4}i + \frac{2}{B^2} \right)\frac{1}{B} \tag{27}$$

$$Z_{\infty} = \frac{1}{A}\,\frac{4B^2}{64 + 9\pi B^2}\,(8 + 3\pi^{\frac{1}{2}}Bi) \tag{28}$$

$$R_\infty = \frac{1}{A}\frac{32}{9\pi}\left(1 - \frac{64}{9\pi B^2}\right) \tag{29}$$

$$X_\infty = \frac{1}{A}\, 12\pi^{\frac{1}{2}} B\left(1 - \frac{64}{9\pi B^2}\right) \tag{30}$$

In Table I the values of $D(x)$ for $B = 1$ are given and compared with the corresponding values for the D. C. case obtained by Spitzer and Härm.[2]

In Table II the resistance, the reactance, and the absolute value of conductivity are given for various B from 0 to 10. For $B > 10$, one may use Equation (26) to compute them. The error will be within 2 per cent. For $B > 50$ the collisions become unimportant and Equations (27) - (30) will give the correct values to within 2 per cent. However, there the validity of the Fokker-Planck Equation already becomes questionable and one should use Dawson-Oberman's values instead of ours.

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Table I

Values of $D(x)$ for $\omega = \omega_c$ and $\omega = 0$

	$\omega = \omega_c$		$\omega = 0$
x	Re(D(x))	Im(D(x))	D(x)*
0.10	0.0005887	-0.0002029	0.0008093
0.11	0.0009252	-0.0003438	0.001300
0.12	0.001376	-0.000542	0.001970
0.13	0.001956	-0.0008068	0.002847
0.14	0.00268	-0.001149	0.003955
0.15	0.00356	-0.00158	0.005317
0.16	0.00461	-0.00210	0.006955
0.17	0.00583	-0.00272	0.008886
0.18	0.00724	-0.00346	0.01113
0.19	0.00884	-0.00431	0.01370
0.20	0.01063	-0.00528	0.01660
0.22	0.01483	-0.00761	0.02347
0.24	0.01985	-0.01048	0.03180
0.26	0.0257	-0.0139	0.04165
0.28	0.0324	-0.0180	0.05304
0.30	0.0400	-0.0226	0.06601
0.32	0.0483	-0.0279	0.08057
0.34	0.0575	-0.0339	0.09672
0.36	0.0675	-0.0405	0.1145
0.38	0.0783	-0.0478	0.1338
0.40	0.0899	-0.0557	0.1548
0.44	0.1153	-0.07366	0.2015
0.48	0.1435	-0.09436	0.2545
0.52	0.1744	-0.1179	0.3137
0.56	0.2080	-0.1442	0.3792
0.60	0.2439	-0.1734	0.4508
0.64	0.2822	-0.2055	0.5285
0.68	0.3227	-0.2405	0.6123
0.72	0.3652	-0.2785	0.7023
0.76	0.4096	-0.3196	0.7983
0.80	0.4559	-0.3637	0.9005
0.88	0.5535	-0.4813	1.123
0.96	0.6570	-0.5718	1.371
1.04	0.7656	-0.6957	1.645

1.12	0.8782	-0.8335	1.945
1.20	0.9937	-0.9858	2.273
1.28	1.111	-1.1531	2.630
1.36	1.2290	-1.3359	3.017
1.44	1.3457	-1.5347	3.435
1.52	1.4598	-1.7500	3.887
1.60	1.5693	-1.9820	4.375
1.76	1.7657	-2.4962	5.465
1.92	1.1915	-3.0739	6.728
2.08	1.9973	-3.7049	8.190
2.24	1.9913	-4.3694	9.880
2.40	1.8852	-5.0382	11.83
2.72	1.4266	-6.2407	16.62
2.88	1.2198	-6.6809	19.53
3.04	1.3105	-6.8066	22.74
3.20	2.1113	-5.6758	26.00

Table II

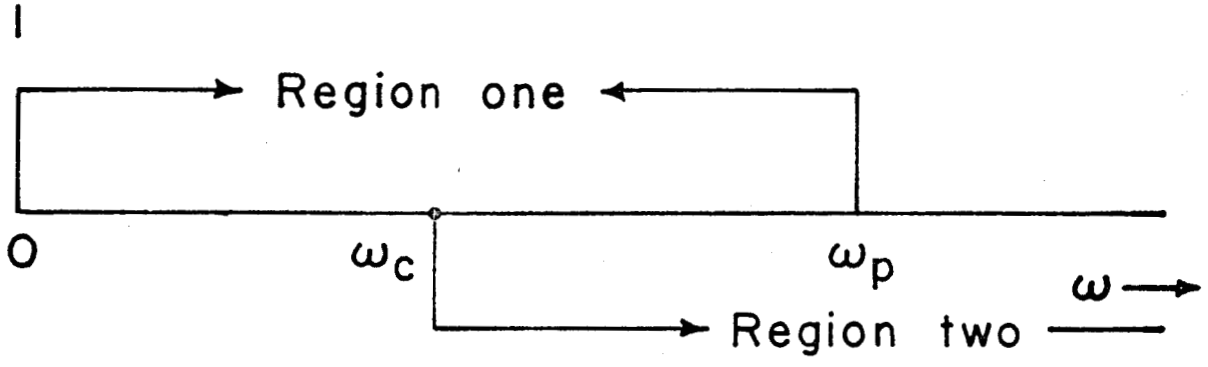
The conductivity, the resistance and

the reactance of A. C. current

	σ/A		R/c		X/c	
ω/ω_c	Real gas	Lorentz gas	Real gas	Lorentz gas	Real gas	Lorentz gas
0.	1.734	3.0	0.577	0.333	0	0
0.05	1.729	2.880	0.577	0.340	0.045	0.070
0.1	1.713	2.653	0.577	0.354	0.089	0.130
0.15	1.687	2.430	0.578	0.367	0.134	0.185
0.2	1.651	2.233	0.579	0.380	0.178	0.236
0.25	1.608	2.061	0.580	0.393	0.223	0.285
0.3	1.561	1.913	0.582	0.404	0.267	0.332
0.35	1.510	1.784	0.584	0.415	0.311	0.377
0.4	1.458	1.671	0.587	0.425	0.354	0.421
0.45	1.406	1.572	0.590	0.434	0.397	0.465
0.5	1.354	1.483	0.593	0.443	0.440	0.508
0.55	1.303	1.405	0.597	0.452	0.482	0.550
0.6	1.255	1.334	0.600	0.460	0.525	0.592
0.65	1.208	1.270	0.603	0.467	0.567	0.634
0.7	1.164	1.212	0.607	0.475	0.608	0.675
0.75	1.122	1.159	0.610	0.482	0.650	0.716
0.8	1.082	1.111	0.614	0.488	0.691	0.756
0.85	1.045	1.066	0.617	0.495	0.732	0.797
0.9	1.009	1.025	0.620	0.501	0.773	0.807
0.95	0.976	0.987	0.624	0.507	0.813	0.877
1.	0.944	0.952	0.627	0.513	0.853	0.916
1.1	0.886	0.889	0.634	0.524	0.934	0.996
1.2	0.834	0.835	0.640	0.534	1.013	1.074
1.3	0.786	0.785	0.645	0.544	1.094	1.159
1.4	0.743	0.741	0.650	0.553	1.172	1.230
1.5	0.721	0.702	0.657	0.561	1.222	1.308
2.	0.560	0.557	0.672	0.599	1.588	1.693
3.	0.408	0.398	0.711	0.655	2.335	2.453
4.	0.313	0.305	0.734	0.696	3.086	3.207
5.	0.255	0.248	0.766	0.728	3.842	3.960
6.	0.214	0.211	0.784	0.754	4.601	4.710
7.	0.183	0.182	0.790	0.776	5.380	5.460
8.	0.162	0.161	0.796	0.794	6.08	6.21
9.	0.144	0.143	0.814	0.810	6.69	6.81
10.	0.130	0.129	0.830	0.824	7.51	7.6

Figure 1

The Ranges of Validity of Computed A.C. Conductivities



The values of A.C. conductivity obtained in this paper is valid in region one. When ω exceeds ω_c -- region two -- the values calculated by Dawson et al begin to be valid.

Here ω_c is the collision frequency, ω_p is the plasma frequency.

Figure 2

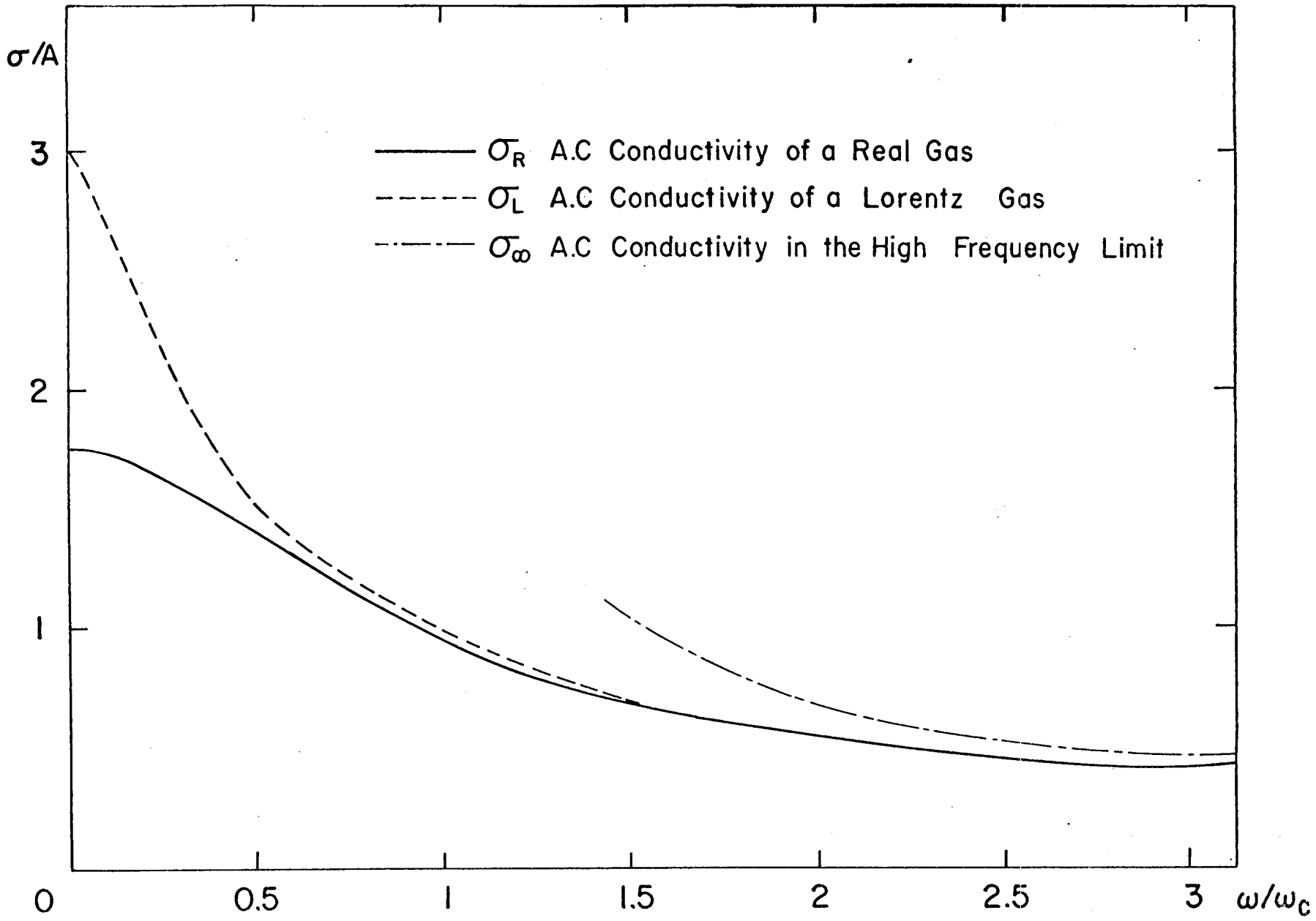

σ/A
3
2
1
0
0.5
1
1.5
2
2.5
3
ω/ω_c
σ_R A.C Conductivity of a Real Gas
σ_L A.C Conductivity of a Lorentz Gas
σ_∞ A.C Conductivity in the High Frequency Limit

Figure 3

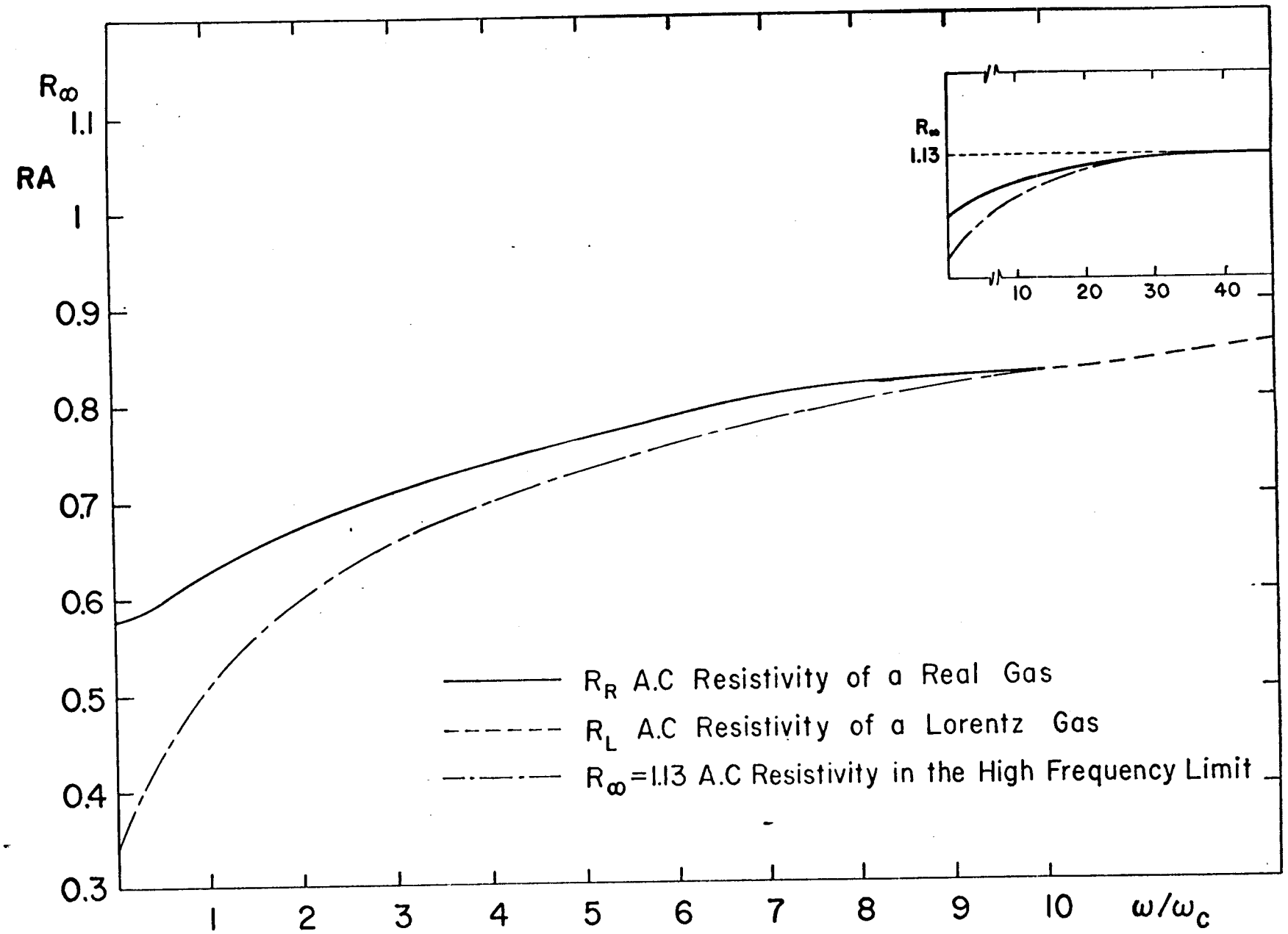
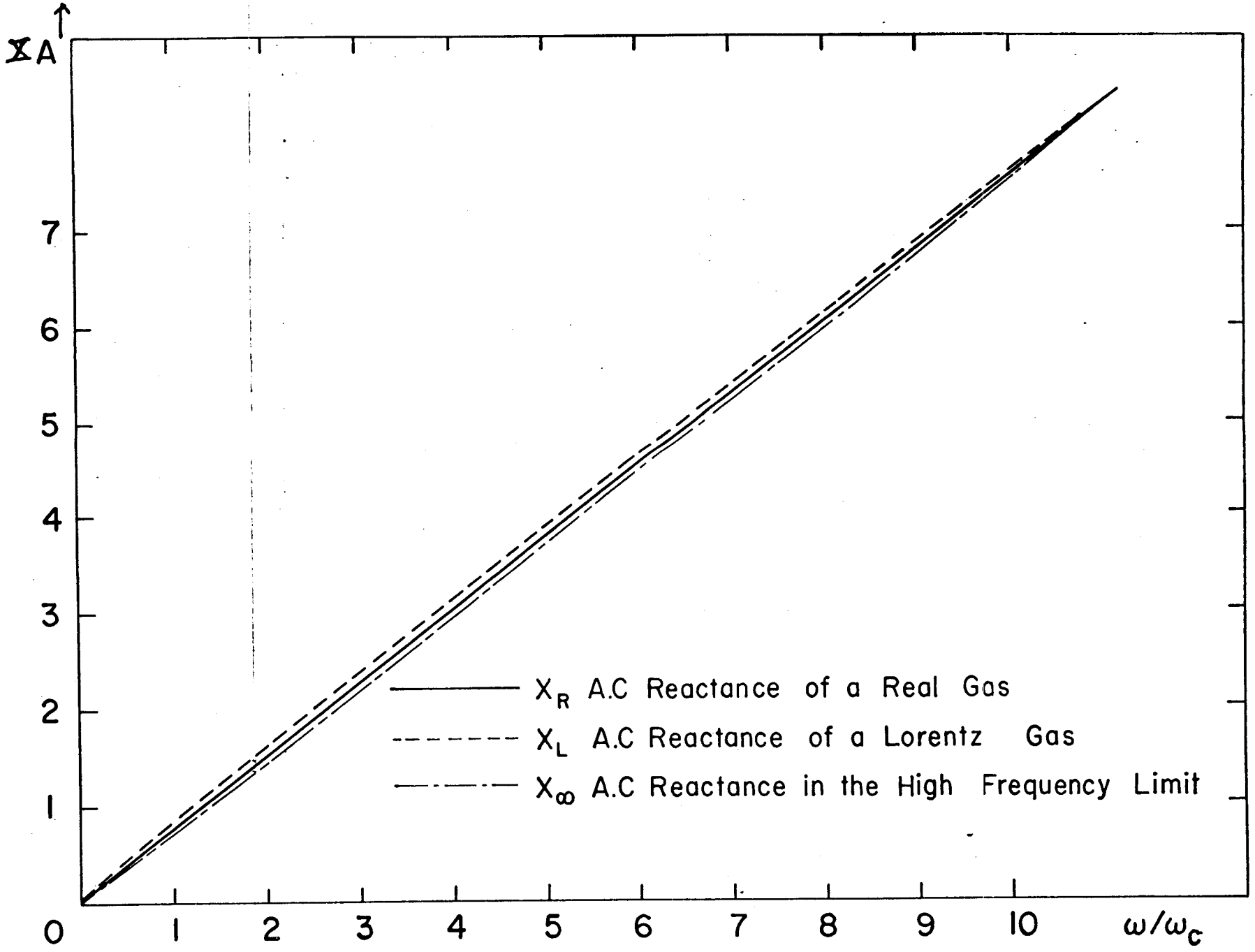

R_∞
1.1
RA
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1
2
3
4
5
6
7
8
9
10
ω/ω_c
R_∞
1.13
10
20
30
40
R_R A.C Resistivity of a Real Gas
R_L A.C Resistivity of a Lorentz Gas
R_∞=1.13 A.C Resistivity in the High Frequency Limit

Figure 4


χ_A
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
ω/ω_c
X_R A.C Reactance of a Real Gas
X_L A.C Reactance of a Lorentz Gas
X_∞ A.C Reactance in the High Frequency Limit